THE ACHIEVEMENTS OF SIR GEORGE EVEREST IN GEODESY

Alan Cook, Selwyn College, Cambridge

1. INTRODUCTION

The geodetic survey of India was begun by Col.Lambton, whose assistant Everest became, at a time when the earliest measurement of a meridional arc in Europe, that from Spain through France to the north of Scotland, had just been completed (Delambre and Méchain, 1821-1845). Geodetic survey is the determination of the coordinates of selected points on the surface of the Earth so that maps may be based on them. Nowadays the Cartesian coordinates of any point may be found by reference to the orbits of artificial satellites, as in the Global Position System, but of course in Everest's day that was far in the future. The surface of the Earth, although irregular, is close to a spheroid of revolution, as Newton shewed theoretically that it should be. To map the surface of the Earth onto a plane sheet of paper, the shape of the actual surface of the Earth has to be known, or, more realistically, the shape of the spheroidal surface that is closest to the actual surface. Geodetic survey must therefore be carried out in such a way as to determine the form of the surface. Geodesy is not however just a matter of geometry. The spheroidal shape of the Earth is a consequence of the distribution of density within the Earth together with the rate at which the Earth rotates upon its polar axis, and is an important datum for studying the physical state and composition of the interior. The deviations from a simple spheroidal shape likewise depend on the distribution of density, but much nearer the surface and quite closely related to the structure of oceans and mountains. Because the value of gravity at the surface depends on the distribution of density within it, measurements of gravity contribute to the estimation of the shape of the surface and to elucidating the causes of the deviations from a spheroidal form.

A geodetic survey is never independent of the value of gravity over the surface. The form of the surface is found from the relation between distance over the surface as measured by triangulation, and the angular coordinates of points on the surface. The only angular coordinates directly open to observation are those of normals to the surface and if the surface of the Earth were an exact spheroid of revolution, then the radius of curvature, the relation between distance s over the surface and latitude ϕ would be:

$$\frac{ds}{d\phi} = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{3/2}}$$

where a is the major semi-axis of the spheroid and e is the eccentricity.

The surface of the Earth is not an exact spheroid and the angles that are observed are those of the directions of the attraction of gravity, that is, normals to the equipotential surface of the gravitational potential through the points of observation. The geodesist determines a spheroid that best fits the observed directions: apart from uncertainties of the actual observations, the differences between observed and calculated angles arise from departures of density within the Earth from a uniform variation with radius.

Newton shewed in the Principia that the surface of a spinning Earth should be an oblate spheroid of revolution and he shewed also that the value of gravity over it should increase from the equator to the poles. When he published the Principia in 1687 there were no survey observations adequate to establish the geometrical form of the Earth, but there were a few observations of gravity (including those of Edmond Halley in St Helena) that agreed with Newton's prediction. Later survey measurements appeared to shew that the Earth had a prolate form, the view of J-D Cassini, but then the French expeditions to Lapland and Peru clearly established the oblate form and in the words of Voltaire, "flattened the Earth and the Cassinis". French academicians went on to survey an arc of the meridian running through France; their work continued even during the Revolution and was extended to the south of Spain and northwards through Great Britain (Delambre and Mechain, 1821-1845). It was supplemented by gravity measurements at a number of principal stations by Arago and Biot in France and Spain, and by Henry Kater in Britain. Those campaigns were essentially completed by about 1820.

The rate of change of the radius of curvature with respect to latitude is zero at the poles and the equator and is greatest in mid-latitudes. Europe is therefore well situated for a determination of the elipticity of the meridian from observations within Europe. At the same time the span of the arc from Spain to Britain is 22 deg which is about one quarter of the whole quadrant of the meridian. A much better determination of the size and eccentricity of the Earth would be obtained if the European results could be combined with those from lower latitudes.

When Col Lambton began the survey of India in 1800 the British and French surveys were the only ones of good accuracy. They owed much to the instrumental developments of Ramsden, whose theodolite was the first that was sufficiently accurate to detect the spherical excess of triangles. Considerable attention was also given to the measurement of base lines and those surveys were the models for the Indian project. By the time Everest had carried the triangulation up to the Himalaya, the results of Struve's survey of an arc of the meridian in the west of Russia were available, so that Everest was able to combine the Indian results with those of two long arcs in much higher latitudes to determine the size and flattening of the Earth.

The Indian surveyors were at a disadvantage compared to their European colleagues in two respects. In the first place, no measurements of gravity were made in India until the work of Basevi and Heaviside fifty or more years after Biot and Kater (Everest took a pendulum out to India but there are no records of its having been used at stations of the Arc).

The second matter relates to the deviations of the true vertical, the actual direction of gravity, from the normal to the best fitting spheroid. The measured astronomical latitude, longitude and azimuth determine the direction of the actual vertical relative to the polar axis of the Earth and the Greenwich meridian. Differences of the angles between normals to an adopted spheroid may be calculated from the elements of the spheroid and the measured distance between the points over its surface. The differences between true directions and calculated directions are connected by a geometrical relation due to Laplace, and points at which astronomical latitude, longitude and azimuth are all observed and compared with the geodetic values derived from the survey are called Laplace points. They are valuable for checking errors of direction that may accumulate in a set of triangles, just as measurements of base lines check the accumulation of errors of scale. However, the measurement of astronomical longitude was difficult prior to the use of the electric telegraph and so there were no Laplace points in the first Indian arc.

2. EVEREST'S ACHIEVEMENTS.

Everest began his geodetic work in India as assistant to Lambton but had to return to England to recuperate on account of ill-health. It seems clear that he intended to return to India and resume the geodetic survey for while in England he arranged for the Indian 10ft standard bar to be compared with the standards of the Ordnance Survey and the Tower of London, as was in fact done after his return to India (Clarke, 1866). He studied the methods and apparatus employed by Colby in Ireland, and he wrote on errors of pendulum observations (Everest 1829). He took over the responsibility for the geodetic survey when he returned to India after Lambton's death; he held in addition the post of Surveyor General. It is clear from his account of his work that he found both staff and methods in poor shape and that he revised the procedures and obtained better equipment in order, as he says, to work to the same standards as the best European practice. He must also have attended carefully to the recruitment and training of his immediate assistants, for while he was clearly dissatisfied with those he found when he returned to India, he was confident enough in his last years to delegate responsibility for substantial independent operations.

Lambton had begun his measurements of the Indian arc in the south in latitude 8deg 9min at Punnae and carried them as far north as 20deg 30min. Everest took them to Kaliana in latitude 29deg 30min, having in the meantime surveyed a parallel eastward to Calcutta. It is ironic that the meridional arc, to which he devoted so much attention and effort, was eventually found to be too disturbed by the attraction of the Himalaya and otherwise, for it to be included in a general world-wide adjustment, whereas the observations on the parallel, somewhat subsidiary in his programme, have proved their value (see Section 3).

Everest suffered from ill health on a number of occasions and on the first of those he was sent to the Cape Colony to recuperate. While there he re-examined the survey of de la Caille which appeared discrepant with surveys elsewhere and concluded that the attraction of Table Mountain had disturbed measurements of latitude nearby (Everest 1822). He later made similar calculations in India to attempt to account for anomalies in the Indian survey. His initial measurement of the arc between Damargida (18deg 3min) and Kalianpu (24eg 7min), with a central astronomical station and base at Takal Khera, appeared to show that the radius of curvature of the meridian in the northern section was less, not greater, than that of the southern section. Everest calculated the attraction of a table-land to the north of Takal Khera and shewed that it could account for the anomaly.

The survey of the Damargida-Kalianpur arc was however as a whole unsatisfactory, with relatively large errors in the sums of the angles of triangles, and Everest therefore repeated it with better instruments, including Colby's compensating bars for base-line measurements. He then extended the arc northward to Kapianpur at 29deg 31min and did not go further because he considered that the disturbance of the Himalayan mountains would be too great.

Everest subsequently repeated some of Lambton's survey to the south and set up new stations where Lambton's could no longer be found. He paid careful attention to the measurement of bases. Astronomical latitudes and longitudes were observed throughout the arc of the meridian, especially at Kalianpur (24deg 7min) where many observations were made.

When he came to publish his final report on the Indian arc, Everest had available the results of a number of other surveys, namely that from Spain through France to Scotland, the arc measured by Struve in western Russian, and the short arc in Peru. He seems to have been the first to drive a figure for the Earth by combining the results of surveys in different parts of the world, as distinct from calculating the radius of curvature in a given latitude from a single survey, so setting the pattern for all future investigations of the size and shape of the Earth. He combined the results by a method that falls short of true least squares, and indeed introduces internal correlations, and obtained the following results (Everest 1847):

Equatorial semi-axis: 3 486 817.08 fm = 20 920 902.48 ft
= 6376 691 m,

Polar semi-axis: 3 475 607.00 fm = 20 853 642.0 ft
= 6356 190 m

The polar flattening is then 1/311.0

Those elements were used for some years for the reduction of Indian surveys and for map projections.

3. THE INFLUENCE OF EVEREST'S WORK

As was pointed out in the Introduction, the Indian arc is important because it lies in much lower latitudes than those of Europe and when combined with them should have enabled much better values for the elements of the figure of the Earth to have been obtained than from any of them separately. The care with which the operations were carried out also seems to have been a model for subsequent work. Everest's own values for the elements of the figure of the Earth are however very different from modern values and indeed from contemporary estimates, and the reasons for that are now considered.

The first reason is that the standard of length for Lambton's southern arc was in error. Only after Everest had sent the Indian standard (B) to the Ordnance Survey Office for comparisons in 1830 was the value of that standard well established, the data for an earlier comparison having been deficient (Everest 1847). In consequence, only the northern section of the whole Indian arc, that for which Everest was entirely responsible, was securely based upon the Ordnance Survey standard. In the course of the extension of the geodetic surveys in India by Everest's successors, the southern section of the arc was revised and referred to the Ordnance Survey standard. In 1866 Clarke published the results of comparisons of the standards of length of countries with major geodetic surveys and also the results of an analysis of all the important data for the figure of the Earth.

Clarke (1880) gives the following values for the equatorial and polar axes, in feet of the British standard yard; Everest's values are those already given.

	Clarke	Everest	Difference
Equatorial axis	2092 6202	2092 0902	5300
Polar axis	2085 4895	2085 3642	1253
Polar flattening	1/293.5	1/311.0	

The second reason for the discrepancy between Clarke's results and those of Everest is that, as Clarke points out, the Indian arc, being in low latitudes, has a strong influence on the estimation of the polar axis but a relatively weak one upon the estimation of the equatorial axis; the radius of curvature of meridian at the equator is $c(1-e^{2})^{\frac{1}{2}}$, whereas at the poles it is $a/(1-e^{2})$.

Later analyses of more recent observations (Jeffreys 1948) have given yet different elements. Jeffreys in fact rejected the data from the Indian meridian because he considered that the uncertainties of the attractions by the Himalayas were too great; his final result was

equatorial semi-axis:	6378 100km,
polar flattening:	1/297.10

Clarke's value of the equatorial semi-axis corresponds to 6378.306km and Everest's to 6376.691km.

The values for the polar axis, the one best determined from the Indian arc, are

Everest:	6356 190 km
Clarke:	6356 572 km
Jeffreys:	6356 632 km

Everest's value for the polar axis is indeed much closer to later estimates than is his value for the equatorial axis, but it is clear that the effective radius of curvature over the Indian meridian is too small.

In the years since Jeffrey's study, triangulation has been superseded or supplemented by direct measurements of length by electromagnetic means and surveys have been adjusted taking into account the effects of variation of the gravitational potential. Most recently, observations to artificial spacecraft have been added. The consequence has been that the following values were derived in 1963 (Cook, 1965) using radar data for the distance of the Moon but not satellite results:

equatorial radius:	6378.144 km
polar flattening:	1/298.26

the latest values, with results from ranging to space craft are

equatorial radius:	6378.137 km
polar flattening:	1/298.257
	(Marsh and others 1989)

The corresponding polar radius is 6356.752 km.

One reason for the lower values that Everest found for the equatorial and polar radii is that the radius of curvature over India is less than the average spheroidal value. Satellite results show that the geoid is depressed by about 80m over most of the sub-continent (Marsh and others 1989) but that does not entirely account for the difference between the Indian survey and others.

Although the Indian meridional survey is now seen to depart appreciably from the mean spheroid Everest's work had a great influence on geodesy by calling attention to the importance of combining surveys made in different places, and especially over different ranges of latitude, if properly representative values of the parameters of the figure of the Earth were to be estimated.

Everest's work was very influential for another reason. The discrepancies between geodetic and astronomical angels in the Indian surveys are not great, implying that the gravitational equipotential surfaces in India are close to those of a common spheroid. Everest had earlier, in 1822, studied the triangulation of de la Caille in Cape Province in south Africa (Everest 1822) and had found that an anomalous result could be accounted for by the attraction of Table Mountain. He consequently expected that the Indian deflexions would be larger, especially in the south where the deficit of mass in the southern seas might have given a deflexion of the vertical to the south; and close to the Himalaya where the attraction of the mountains might again have deflected the vertical to the south. The deflexions at those extremities were in fact little more that 3 sec.

The explanation for the small deflexions was provided by Archdeacon Pratt of Calcutta who shewed that if the average density of material beneath the Himalayan mountains was less than that below the Indo-Gangetic plain, the net deflexion of the vertical would indeed be quite small. That was the first indication of the principle of isostasy whereby extra mass above sea level, as in high mountains, is compensated by a corresponding deficit below sea level. Similarly, the lower mass of the water of the oceans is compensated by extra mass below them. Sir George Airy devised a somewhat different scheme of compensation from that proposed by Pratt and later observations of gravity in India and survey operations in the mountainous regions of the United States amply confirmed the prevalence of isostatic balance. Gravity measurements at sea in the last half of this decade have shewn that over very large areas of the the Earth, isostatic balance is maintained to within about twenty parts in a million of the attraction of gravity, or about one part in twenty of the difference of attraction between oceans and continents. Isostatic balance is one of the most important features of the structure of the outermost parts of the Earth and the ways in which it comes about, not yet fully understood, are closely related to tectonic processes in general. Everest's surveys were the first to bring out clearly the existence of isostasy, for the European surveys covered ground with comparatively minor tectonic features which would not in any case cause great deflexions of the vertical.

4. CONCLUSION

Everest's work in India was seen at the time, and has been recognised ever since, as major advance in geodesy, both in applying the most precise methods and apparatus of the day outside Europe and by recognising the world-wide scope of geodesy through his combination of results from a number of meridional arcs. The Royal Astronomical Society awarded him a testimonial, equivalent to the Gold Medal of the Society, and in presenting it, the President of the Society, Sir John Herschel, said

> "The Great Meridional Arc of India is a trophy of which any nation, or any Government of the world, have reason to be proud, and will be one of the most enduring monuments to their power and enlighten regard for the progress of human knowledge."

So it has proved to be.

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