

MEMORANDUM, showing the final result of Archdeacon Pratt's calculations regarding the effect of Local Attraction upon the operations of the Great Trigonometrical Survey of India.

To the Secretary of the Asiatic Society of Bengal.

DEAR SIR,—Having now received from London some copies of the last of my communications to the Royal Society on the amount of local attraction in India and its effect on the operations of the Trigonometrical Survey, I beg to present to the Asiatic Society a complete set of my papers on this subject bound up in one volume, and to request you to give insertion in your Journal to the following Memorandum, which gives a brief history of the circumstances connected with this investigation and of its final results.

I am, yours faithfully,

Calcutta, April 30th, 1862.

JOHN H. PRATT.

MEMORANDUM.

The influence of Mountain Attraction upon the position of the plumb-line and of the spirit-level in the operations of the Great Trigonometrical Survey of India was first pointed out to me by the Surveyor General in 1852, who on that occasion requested me to turn my attention to the subject. The result has been a series of papers which have been published in the Transactions of the Royal Society for 1854, 1855, 1858 and 1861. During the nine years over which the investigation has extended, new information has been obtained from time to time, and new suggestions have presented themselves to my mind. Some things which had been published in one paper have had to be modified in a subsequent one, and the object of this Memorandum is, now that the series is complete, to state what is the final result of the investigation.

2. I will give a brief historical sketch of the circumstances connected with the publication of the successive papers in the *Philosophical Transactions*.

The Surveyor General of India pointed out to me in 1852, that in the volume published by his predecessor Colonel Everest in 1847, giving an account of the measurement of the two northern portions of the Great Arc between Kaliana and Kalianpoor, and Kalianpoor and Damagida, lying in the longitude of Cape Comorin, the observed

or astronomical amplitudes* were, the one 5".236 less and the other 3".791 greater than the calculated or geodetic amplitudes, the curvature of the Indian Arc being taken the same as that of the mean figure of the earth. This discrepancy was supposed to arise from local attraction,† deranging the position of the vertical determined by the plumb-line. This was a highly probable conjecture: but it required demonstration. The problem, then, which I set myself to solve was, To calculate by some direct method the actual amount of the attraction of the Himalayan mass, and of the deflection caused by it in the plumb-line. The result is shown in the FIRST PAPER of the series, *Phil. Trans.* 1854, p. 85, art. 43, (see also *Phil. Trans.* for 1858, p. 769, art. 22 of the Second Paper). The result therein obtained is very much larger than was expected or was required to explain the differences in the astronomical and geodetic amplitudes which Colonel Everest had detected. This calculation seemed, therefore, to increase the difficulty which it was intended to remove; as, in the course of the investigation, this new fact came out, that the disturbing effect of the Himalayas is far greater in amount than any one had ever anticipated, and also of far more extensive influence, as its amount in the centre of India is found to be greater than it was supposed to be even at Kaliana only sixty miles from the hills.

To meet this new difficulty, Mr. Airy, the Astronomer Royal, suggested that there is probably a deficiency of matter immediately beneath the mountains, such as to counteract their effect upon stations in the plains. He assigns his reasons in a paper published in the same volume of the *Philosophical Transactions* and which I have introduced in this series for convenience of reference, (pp. 101—104) Objections to this hypothesis are given in the postscript to a paper I wrote on the English Arc in the volume for 1855, and which is also introduced on account of that postscript, (see p. 51).

* For the benefit of non-Scientific readers I will mention that the *amplitude* of an arc of meridian is the difference of latitude of its extremities.

† If the earth were a perfect spheroid and its materials as we descend downwards were arranged in concentric spheroids, such as the mass would assume if it were fluid, then the total attraction of the earth's mass at any point of its surface would be perpendicular to the surface and the plumb-line would hang in that perpendicular. But if there were any superficial masses, such as mountains, or hollows, such as oceans, or any defect or excess of density in any parts of the earth's crust, a corresponding change would take place in the total amount and direction of the attraction. The resultant effect of these new and disturbing causes at any place is called the LOCAL ATTRACTION at that place.

Four years after this, following up Mr. Airy's suggestion, I proposed and reduced to calculation another hypothesis regarding deficiency of matter below the mountains; viz. that the irregularities of the mountain surface have arisen from the expansion upwards of the crust of the earth from depths below, which has upheaved the mountains and produced a slight but extensive attenuation of the mass below them. The result of this calculation is given in the SECOND PAPER of this series. I show that it is sufficient to produce a considerable amount of compensation for mountain attraction; but that it does not clear up the difficulties; and that as this attenuation is a mere hypothesis, nothing certain can be determined regarding it.

In this same paper it is shown that a very slight but wide-spread defect or excess of density in the materials of the crust of the earth is capable of producing a sensible and important effect on the plumb-line. Thus the possible and not improbable existence of an unknown cause of derangement of the plumb-line hitherto unthought of, as being hidden in the crust, was brought to light.

During the same year it occurred to me that there is another visible cause of disturbance besides the mountains which might produce a sensible effect, viz. the ocean, as its density is less than that of rock. In the THIRD PAPER this effect is calculated, and found to be of importance: (see *Phil. Trans.* for 1858, p. 790, art. 11). Thus a new source of error was detected.

3. Thus far, then, the attempt to clear up the discrepancies detected in the first instance by Colonel Everest between the astronomical and geodetical amplitudes had led to the discovery, that (1) the Himalayas attract places in the plains of India with a force far greater in amount than any person had conceived: And not only so, but that (2) the ocean also has an important influence of the same kind: And more than this, that (3) variations of density in the crust, which are as likely to exist as not, will produce the same effect.

The uncertainty, as to the form of the Himalayas and the depth of the ocean, produces a corresponding degree of uncertainty as to the exact amount of the attraction; while our utter ignorance regarding the condition of the crust below seemed to leave us in hopeless perplexity regarding the derangement which may proceed from that quarter. So that the attempt to determine the resultant amount of local attraction at stations on the Indian Arc by direct calculation would appear, for these reasons, altogether fruitless.

As noticed in these papers, I conceived also that the difference between the geodetic and astronomical amplitudes might arise, not solely from attraction influencing the plumb-line, but in part from the curvature of the Indian Arc being somewhat different from the curvature of the mean figure of the earth. Geology teaches us, that the earth's surface has undergone changes of level. The surface, therefore, cannot be now an exact spheroid. In this case the normals at the extremities of the actual arc would include an angle not precisely equal to the amplitude of the mean or undisturbed arc, and part of the errors to be accounted for might, it was thought, arise from this; the remainder arising from local attraction influencing the plumb-line, and therefore affecting the observed or astronomical amplitude. This served to introduce a new element of difficulty.

4. The ambiguity, however, with which the question was thus beset from all these causes is removed in the FOURTH PAPER, the last of the series, published in the *Philosophical Transactions* of 1861. The following theorem is there demonstrated:—That the length of the actual arc, altered as its form and position may be by geological changes, is nevertheless sensibly equal to the length of the mean or undisturbed arc. Hence, if we calculate the amplitude by using the measured length of the arc, and the *mean axes*, as is done in the Survey, it will come out the mean or undisturbed amplitude. The consequence of this is, that the relative position of places laid down on a map from geodetic operations is correct, and free from all sensible error arising from local attraction, from whatever causes local attraction may arise.

This is a most important practical result, and frees the Survey operations from a doubt which has attached to their high scientific accuracy, ever since it has been discovered that the influence of the Himalayas and of the ocean is so considerable, and that variations in the earth's crust below may have an important disturbing effect. This theorem, moreover, gives us a direct means of estimating at once the difference of local attractions, and of local deflections caused by them, at the extremities of an arc. For the difference is precisely equal to the quantity by which the astronomical amplitude differs from the mean or undisturbed amplitude found as above described.

5. There is only one desideratum remaining; but one which I

fear will never be met ; that is, To devise a method for determining the absolute latitude of some one place included in the map. The state of the question is, as I have said, at present *this* : the position of places determined by geodetic operations is correct and free from the effect of local attraction, *relatively* to the station from which the operations start. But how to find the latitude of this starting point, freed from the errors produced by local attraction, is a problem unsolved, and unlikely to be solved. Even if any spot exists which is altogether free from local attraction, that is where all such influences nullify each other, it is impossible to discover it and to assure ourselves of the fact.

6. Thus geodesy can give us accurate maps of the relative position of places ; but cannot, with the same accuracy, assign the position of the maps on the terrestrial spheroid. Suppose, to take a comprehensive case, that the whole globe were surveyed and all places in it connected by triangulation with the spot in the north where the plumb-line points to the north-pole in the heavens. The positions of all places would be found free from error relatively to this spot—which is commonly called the North Pole of the earth. But how can we be sure that the plumb-line at that spot is hanging in the true vertical ? It may be under the influence of local attraction : in which case, although it points to the pole in the heavens, the spot in question will not be the pole on the earth. There is no means, nor can I conceive any means possible, short of ascertaining all the disturbing causes throughout the earth's mass and calculating their effects, of determining whether the plumb-line *is* or *is not* at the true pole. The accurate position, therefore, of our maps on the terrestrial spheroid which depends upon this question is alike unknown and uncertain. This is the point to which the investigation is brought, and where, I have no doubt, it will stop. It is satisfactory that the mapping of a country may be laid down, free from all error as to the relative situation of places : also that the relative amount of local attraction, comparing one place with another, can be determined, because this may assist in ascertaining the structure of the crust below. It would, however, be still more satisfactory if this one remaining difficulty could be removed, as it would make the data more complete for the high scientific determination of the Figure of the Earth.